BRIGHT EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the* eight *questions in section* A *and any* **five** *questions from section B.*

*Any additional question(s) answered will **not** be marked.*

All *necessary working **must** be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

*Answer **all** the questions in this Section.*

1.   Evaluate: $\int_e^{e^3} \frac{dP}{P(lnP)^2}$ *(5 marks)*

2.   Use small changes to estimate $tan48^0$, correct to 2 decimal places.

                                                                                                                           (5 marks)

3.   Prove that $2\log_c(a+b) = 2\log_c a + \log_c\left(1 + \frac{2b}{a} + \frac{b^2}{a^2}\right)$.

                                                                                                                           (5 marks)

4.   Solve the simultaneous equations: $2x^2 - 5xy + 2y^2 = 0$
                                                                    $x + y = 6$

                                                                                                                           (5 marks)

5.   Given that $Z = \frac{(1-i)(-8i+6)}{(4+3i)}$, express Z into polar form.   *(5 marks)*

6.   The distance of the point $(2, -1)$ from the line $y = \frac{3}{4}x + P$ is twice its distance from the line $y = \frac{-3}{4}x$. Find the value of P.

                                                                                                                           (5 marks)

7.   Find the equation of the curve whose gradient is given by e^{x-y} and passes through point $(0, ln2)$.

                                                                                                                           (5 marks)

8.   An arithmetic progression has n terms. The sum of the first three terms is 15 and the sum of the last three terms is 168. If the tenth term is 29, find the value of n.

                                                                                                                           (5 marks)

*Answer any **five** questions from this Section.*

***All** questions carry equal marks.*

9. A particle moves in a straight line such that its velocity in m/s after passing through a fixed point **O** is $3cost - 2sint$. Find the:

   (a) Its distance from **O** after $\frac{\pi}{2}$ seconds. *(3 marks)*

   (b) Its acceleration after π seconds. *(2 marks)*

   (c) The time when its velocity is first **zero**. *(7 marks)*

10. Express $\frac{2x^2-9x+7}{(x-3)^2}$ as a **sum** of its partial fractions.

Hence show that $\int_4^5 \frac{2x^2-9x+7}{(x-3)^2} dx = \ln(8e)$. *(12 marks)*

11. (a) Prove that $\frac{cos3A}{cosA} - \frac{sin3A}{sinA} = 4cos2A$ *(4 marks)*

   (b) Show that $tan^{-1}\left(\frac{1}{3}\right) + sin^{-1}\left(\frac{1}{\sqrt{5}}\right) = \frac{\pi}{4}$. *(8 marks)*

12. (a) The point C divides the line $\overline{AB}$ in the ratio of 1:2 and the position vectors of A and C are $(-4i - 3j + 5k)$ and $(3i - 2j + 12k)$ respectively. Find the co – ordinates of B. *(5 marks)*

   (b) A plane contains points $K(4,-6,5)$ and $L(2,0,1)$. A perpendicular line from the point $P(0,4,-7)$ meets this plane at point M. find the Cartesian equation of line $\overline{PM}$. *(7 marks)*

13. Given the curve $y = \frac{(x-2)^2}{(x+1)}$.

   (a) Find the turning point and its nature. *(4 marks)*

   (b) Determine the region where the curve does not exist. *(2 marks)*

   (c) State the asymptotes and intercepts hence sketch the curve.

                                                                    (6 marks)

14. (a) Expand $\left(\frac{1-x}{1+2x}\right)^{\frac{1}{4}}$ up to the **fourth** term. *(6 marks)*

(b) The roots of the quadratic equation $3m^2 - 5m + 4 = 0$ are $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. Form a quadratic equation whose roots are $\frac{\alpha}{\beta^2}$ and $\frac{\beta}{\alpha^2}$. *(6 marks)*

15. (a) Show that the equation $25x^2 + 9y^2 - 100x - 54y = 44$ represents an ellipse. State the co – ordinates of its center and focus. *(4 marks)*

(b) Determine the equation of the normal to the circle that passes through points (5,0), (6,0) and (8,6) at point (2,0). *(8 marks)*

16. At **2:00PM**, a hot liquid was found on a table in a kitchen of room temperature 15^0C. The initial temperature measurement was 85^0C but 5 minutes later, its temperature had dropped by 25^0C from the initial temperature. It was found out that the rate at which the liquid lost heat was proportional to the difference of the temperature of the liquid and that of the room. If the normal temperature of this liquid is 100^0C, estimate the time for which the hot liquid might have been put on the table.

(12 marks)

END